INTEGRAL CHARACTERISTICS OF MHD CHANNEL WITH NONCONDUCTING BAFFLES

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The electromagnetic characteristics of an MHD channel with insulated walls, two quite long electrodes, and nonconducting baffles are obtained for arbitrary magnetic field distribution law along the channel. Some specific cases of magnetic field and baffle location specification are examined in detail.

1. We consider a flat channel $|x| < \infty$, $0 < y < H$ with two semi-infinite electrodes $\mu < x < \infty$, $y = 0$ and $y = H$. Into the channel there are introduced $n-1$ infinitely thin nonconducting baffles whose left ends extend to infinity, and the coordinates of the right ends are $(0, kH/n)$ $(k = 1, \ldots, n-1)$ (Fig. 1a).

We assume that the electrical conductivity of the medium is constant, the velocity distribution is given in the form $V = (V, 0, 0)$ $(V = \text{const})$, and the magnetic field is expressed by the relation

$$\mathbf{B} = (0,\ 0,\ -B(x)) \qquad (1.1)$$
$$B(x) = B_0 b(x),\ b(x) \to 0 \text{ for } x \to -\infty$$

where B_0 is the magnetic field magnitude characterizing the problem. Then, for small magnetic Reynolds numbers we must solve the following system of equations and boundary conditions to find the electrical characteristics of the channel [1]:

$$j_x = -\sigma \frac{\partial \varphi}{\partial x},\ j_y = -\sigma \frac{\partial \varphi}{\partial y} + \sigma V B(x), \quad \nabla^2 \varphi = 0$$
$$\partial\varphi / \partial x = 0 \text{ for } |x| \to \infty,\ \partial\varphi / \partial y = 0 \text{ for } x = -\infty \qquad (1.2)$$
$$\varphi = {}^1/_2\, U \text{ for } y = H,\ \mu < x < \infty$$
$$\partial\varphi / \partial y = U / H \text{ for } x = \infty,$$
$$\varphi = -{}^1/_2\, U \text{ for } y = 0,\ \mu < x < \infty,$$

$\partial\varphi/\partial y = VB(x)$ on the insulated walls and baffles.

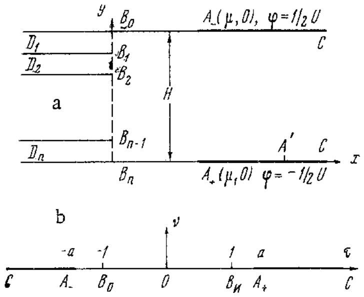

Fig. 1

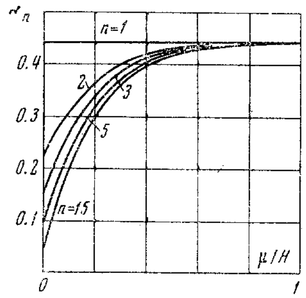

Fig. 2

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Here j is the current density, φ is the electric potential, and U is the voltage on the electrodes.

If we now introduce the analytic function

$$f(z) = p + iq = \frac{\partial\varphi}{\partial y} + i\frac{\partial\varphi}{\partial x} \tag{1.3}$$

the problem (1.2) reduces to finding this function, satisfying the boundary conditions: $q = 0$ at the electrodes; $p = VB(x)$ at the insulated baffles and walls. To solve the posed problem we map conformally the strip $|x| < \infty$, $0 < y < H$ with cuts along the baffles onto the upper half-plane of the $t = \tau + i\nu$ plane (Fig. 1b) with the aid of the formula [2]

$$z = H/n\pi \ln T_n(t), \quad T_n(t) = {}^1/_2[(t + \sqrt{t^2-1})^n + (t - \sqrt{t^2-1})^n]; \tag{1.4}$$

$T_n(t)$ are the Chebyshev polynomials. In this case the points B_k and D_m of the z plane will correspond to the points $(\tau_k, 0)$ and $(\tau_m, 0)$ of the t plane, where

$$\tau_k = \cos\theta_k, \quad \theta_k = -\frac{\pi}{2n} + \frac{k\pi}{n}, \quad T_n(\tau_k) = 0 \quad (k = 1, \ldots, n) \tag{1.5}$$

$$\tau_m' = \cos\theta_m', \; \theta_m' = \frac{m\pi}{n}, \; \frac{dT_n(\tau_m')}{d\tau} = 0 \quad (m = 1, \ldots, n-1)$$

The correspondence of the other points is clear from Fig. 1, and the quantity a is found from the equation

$$T_n(a) = \exp(\mu n\pi / H) \tag{1.6}$$

The analytic function $f(z)$ (1.3) in the region of the complex variable t becomes the analytic function

$$f_1(t) = p_1 + iq_1 \tag{1.7}$$

satisfying the boundary conditions

$$f_1(\tau_k) = 0, \; f_1(\infty) = U / H$$

$$p_1 = VB(\tau) \text{ on } A_-A_+, \; -a < \tau < a, \; q_1 = 0 \text{ on } CA_- \text{ and } A_+C, \; a < |\tau| < \infty \tag{1.8}$$

Thus we obtain a mixed boundary value problem for the harmonic function f_1, where f_1 must have singularities of the pole type at the points τ_m', corresponding to the ends of the baffles, since at these points the current density becomes infinite ($q = \infty$, while $p = VB(x)$ remains finite). As is done in [3] in deriving the Keldysh–Sedov formula, it can be shown that the posed problem is solved by the formula

$$f_1(t) = \frac{VB_0}{i\pi g(t)} \int_{-a}^{a} \frac{g(\tau)\,b(\tau)\,d\tau}{\tau - t} + \frac{U}{Hg(t)} + \frac{VB_0}{\Pi(t)}\left(\gamma_0 + \sum_{m=1}^{n-1} \frac{\gamma_m}{t - \tau_m'}\right) \tag{1.9}$$

$$g(t) = \sqrt{(t-a)/(t+a)}, \quad \Pi(t) = \sqrt{(t-a)(t+a)}$$

Here we examine that branch of the root which is positive on the segment (a, ∞). It follows from the condition $f_1(\tau_k) = 0$ that the constants γ_0, γ_m satisfy the system of equations

$$\gamma_0 + \sum_{m=1}^{n-1} \frac{\gamma_m}{\tau_k - \tau_m'} = (J - K)(a + \tau_k) + F_k \tag{1.10}$$

$$J = \frac{1}{\pi}\int_0^\pi b(\vartheta)\,d\vartheta, \quad F_k = \frac{a \sin\vartheta_k}{\pi}\int_0^\pi b(\vartheta)\,\mathrm{ctg}(\vartheta - \vartheta_k)\,d\vartheta \tag{1.11}$$

$$K = U / E, \; E = HB_0V, \; \cos\vartheta = \tau / a, \; \cos\vartheta_k = \tau_k/a, \; b(\vartheta) \equiv b[x(\vartheta)]$$

Here K is the load coefficient. In deriving the formula for F_k, we used the identity

$$\frac{1}{\cos\psi - \cos\chi} = \frac{1}{2\sin\chi}\left(\mathrm{ctg}\frac{\chi + \psi}{2} + \mathrm{ctg}\frac{\chi - \psi}{2}\right) \tag{1.12}$$

Using the symmetry of the location of the points τ_k and τ_m' relative to $\tau = 0$ (see (1.5)), and also the the equality $F_k(\tau_k) = -F(-\tau_k)$, we can show easily that

$$\gamma_0 = (J - K)a \tag{1.13}$$

2. To determine the electrical characteristics of the channel we must know the electromagnetic power developed in the channel and the current flowing to the electrodes. Since in our case the electrodes are considered to be infinitely long, these quantities will also be infinitely large. On the other hand, such a channel can be represented as half of a channel with finite (length 2λ) but very long electrodes, the current I of which equals $I = 2I_\lambda$, where I_λ is the current to the part of the electrode of length λ of the subject channel (segment A_+A' in Fig. 1a) for sufficiently large λ/H. Now let us find the current I_λ as $\lambda/H \to \infty$. From (1.2), (1.3), and (1.7), we have

$$\frac{I_\lambda}{\sigma} = \int_\mu^{\lambda+\mu} \left(-\frac{\partial\varphi}{\partial y} + VB\right) dx = VB_0 \int_\mu^{\lambda+\mu} b(x)\,dx - \frac{H}{n\pi}\int_a^A p_1(\tau, 0)\, d\ln T_n(\tau); \tag{2.1}$$

the quantity A is found from the equation

$$T_n(A) = \exp\left[(\lambda + \mu)n\pi / H\right] \quad (A \to \infty \text{ for } \lambda/H \to \infty) \tag{2.2}$$

It follows from (1.9) that

$$\frac{p_1(\tau, 0)}{B_0 V} = \frac{1}{\pi}\left(\frac{\tau + a}{\tau - a}\right)^{1/2} \int_{-a}^{a} \left(\frac{a - \tau_1}{a + \tau_1}\right)^{1/2} \frac{b(\tau_1)\, d\tau_1}{\tau_1 - \tau} + K\left(\frac{\tau + a}{\tau - a}\right)^{1/2} + \frac{1}{\sqrt{\tau^2 - a^2}}\left(\gamma_0 + \sum_{m=1}^{n-1} \frac{\gamma_m'}{\tau - \tau_m'}\right) \tag{2.3}$$

With account for (1.13), (2.3) can be rewritten as

$$\frac{p_1(\tau, 0)}{B_0 V} = \frac{1}{\sqrt{\tau^2 - a^2}}\left[(K - J)\tau + \sum_{m=1}^{n-1} \frac{\gamma_m'}{\tau - \tau_m'}\right] + \frac{\sqrt{\tau^2 - a^2}}{\pi a}\int_0^\pi \frac{b(\vartheta)\, d\vartheta}{(\tau / a) - \cos\vartheta} + \frac{1}{\sqrt{\tau^2 - a^2}} \sum_{m=1}^{n-1} \frac{\gamma_m''}{\tau - \tau_m'} \tag{2.4}$$

$$\gamma_m = \gamma_m' + \gamma_m''$$

$$\sum_{m=1}^{n-1} \frac{\gamma_m'}{\tau_k - \tau_m'} = (J - K)\tau_k \tag{2.5}$$

$$\sum_{m=1}^{n-1} \frac{\gamma_m''}{\tau_k - \tau_m'} = F_k \tag{2.6}$$

Let us show that the expression in the brackets in (2.4) can be written in the form

$$[\ \ ] = (K - J)\frac{nT_n(\tau)}{T_n'(\tau)} \tag{2.7}$$

Actually

$$[\ \ ] = (K - J)\, M_n(\tau) \prod_{m=1}^{n-1} \frac{1}{\tau - \tau_m'}$$

where $M_n(\tau)$ is a polynomial of degree n, satisfying in accordance with (2.5) the condition $M_n(\tau_k) = 0$. Consequently $M_n(\tau) = 2^{1-n}T_n(\tau)$, and since

$$\frac{dT_n(\tau)}{d\tau} = n2^{n-1} \prod_{m=1}^{n-1} (\tau - \tau_m')$$

(2.7) is proved.

Substituting (2.4) into (2.1) with account for (2.7), we obtain

$$\frac{2I_\lambda}{\sigma E} = \Phi(J - K) - I_1 - I_2 + \frac{2}{H}\int_\mu^{\lambda+\mu} b(x)\, dx \tag{2.8}$$

where

$$I_1 = \frac{2}{n\pi}\sum_{m=1}^{n-1} \gamma_m'' \int_a^a \frac{d\ln T_n(\tau)}{(\tau - \tau_m')\sqrt{\tau^2 - a^2}}, \quad \Phi = \frac{2}{\pi}\int_a^A \frac{d\tau}{\sqrt{\tau^2 - a^2}}, \quad I_2 = \frac{2}{n\pi^2 a}\int_0^\pi d\vartheta\, b(\vartheta) \int_a^A \frac{\sqrt{\tau^2 - a^2}\, d\ln T_n(\tau)}{(\tau / a) - \cos\vartheta}$$

For large values of λ/H

$$\Phi = c + \alpha_n, \quad \alpha_n = \frac{2\ln 2}{n\pi} + \frac{2\mu}{H} - \frac{2}{\pi}\ln a, \quad c = \frac{2\lambda}{H} \tag{2.9}$$

Here c is the channel characteristic ratio, and Φ is the inverse of the dimensionless integral resistance of the channel in the case in which the fluid is at rest, which follows from (2.8) if we set $V=0$. The dependence of α_n on μ/H, constructed using (2.9), is shown in Fig. 2 for the values n=1, 2, 3, 5, 15. Now let us examine the integral I_1. Since

$$d\ln T_n(\tau) = \sum_{k=1}^{n}\frac{d\tau}{\tau-\tau_k} \tag{2.10}$$

then

$$I_1 = \frac{2}{n\pi}\sum_{k=1}^{n}\sum_{m=1}^{n-1}\gamma_m''\int_a^A \frac{d\tau}{(\tau-\tau_m')(\tau-\tau_k)\sqrt{\tau^2-a^2}} = \frac{2}{n\pi}\sum_{k=1}^{n}F_k\int_a^A\frac{d\tau}{(\tau-\tau_k)\sqrt{\tau^2-a^2}}$$

This follows from (2.6) and the relations

$$\frac{1}{(\tau-\tau_m')(\tau-\tau_k)} = \frac{1}{\tau_m'-\tau_k}\left(\frac{1}{\tau-\tau_m'} - \frac{1}{\tau-\tau_k}\right), \quad \sum_{k=1}^{n}\frac{1}{\tau_m'-\tau_k} = 0$$

Taking the integral and substituting the value of F_k from (1.11), we obtain

$$I_1 = \frac{2}{n\pi^2}\sum_{k=1}^{n}(\pi-\vartheta_k)\sin\vartheta_k\int_0^{\pi}\frac{b(\vartheta)\,d\vartheta}{\cos\vartheta_k-\cos\vartheta} \tag{2.11}$$

We obtain the expression for I_2 similarly:

$$I_2 = J\Phi + \frac{2}{n\pi^2}\sum_{k=1}^{n}\int_0^{\pi}\frac{(\pi-\vartheta)\sin\vartheta b(\vartheta)\,d\vartheta}{\cos\vartheta_k-\cos\vartheta} - \frac{2}{n\pi^2}\sum_{k=1}^{n}(\pi-\vartheta_k)\sin\vartheta_k\int_0^{\pi}\frac{b(\vartheta)\,d\vartheta}{\cos\vartheta_k-\cos\vartheta} \tag{2.12}$$

Substituting (2.11) and (2.12) into (2.8) with account for (1.23), we find the final expression for the current taken from the electrodes

$$\frac{I}{\sigma E} = cG_1 - K\Phi + \beta_1, \quad G_1 = \lambda^{-1}\int_{\mu}^{\lambda+\mu} b(x)\,dx \tag{2.13}$$

$$\beta_1 = \frac{4}{n\pi^2}\int_0^a \arcsin\frac{\tau}{a}\, b(\tau)\,d\ln T_n(\tau) \tag{2.14}$$

3. Now let us find the electromagnetic power developed in the channel

$$P_2 = 2\int_{-\infty}^{\lambda+\mu}dx\int_0^H dy\,(\mathbf{B}\times\mathbf{j})\,\mathbf{V} = 2\sigma V^2B_0^2H\int_{-\infty}^{\lambda+\mu}b^2(x)\,dx$$
$$-\,2\sigma VB_0U\int_{\mu}^{\lambda+\mu}b(x)\,dx + 2\sigma VB_0\int_S b(x)\varphi(x)\,dx \tag{3.1}$$

Here the index S denotes integration over the insulated walls and both sides of the baffles, and $\varphi(x)$ in this integral is found from the relation

$$\varphi(x) = -\frac{U}{2} - \frac{H}{n\pi}\int_{\tau(x)}^{a} q_1(\tau, 0)\,d\ln T_n(\tau) \tag{3.2}$$

Substituting (3.2) into (3.1), we can obtain

$$\int_S = \frac{H^2VB_0}{n^2\pi^2}\int_0^{\pi} b(\vartheta)J(\vartheta)\,d\ln T_n(\vartheta), \tag{3.3}$$

where

$$J(\vartheta) = \int\limits_{a\cos\vartheta}^{a} q_1(\tau, 0)\, d\ln T_n(\tau)$$

$$\frac{q_1(\tau, 0)}{B_0 V} = \frac{(\tau + a)(J - K)}{\sqrt{a^2 - \tau^2}} + \frac{\sqrt{a^2 - \tau^2}}{\pi} \int\limits_0^{\pi} \frac{b(\vartheta)\, d\vartheta}{\tau - a\cos\vartheta} - \frac{1}{\sqrt{a^2 - \tau^2}} \left(\gamma_0 + \sum_{m=1}^{n-1} \frac{\gamma_m}{\tau - \tau_m'}\right)$$

Using (2.5)-(2.7), we have

$$J(\vartheta) = n(J - K)\vartheta - \sum_{m=1}^{n-1} \gamma_m'' \int\limits_{a\cos\vartheta}^{a} \frac{d\ln T_n(\tau)}{(\tau - \tau_m')\sqrt{a^2 - \tau^2}} - \frac{1}{\pi}\int\limits_0^{\vartheta} \sin\vartheta\, d\ln T_n(\vartheta) \int\limits_0^{\pi} \frac{b(\vartheta')\, d\vartheta'}{\cos\vartheta - \cos\vartheta'} \tag{3.4}$$

Just as was done with the integral I_1 of Section 2, it can be shown that the second term in the right side of the expression for $J(\vartheta)$ equals

$$\sum_{k=1}^{n} \frac{F_k}{a} \int\limits_0^{\vartheta} \frac{d\vartheta'}{\cos\vartheta' - \cos\vartheta_k}$$

Using (2.10), the last term in (3.4) together with the expression $n\vartheta J$ can be represented as

$$\frac{1}{\pi}\int\limits_0^{\vartheta} d\vartheta \int\limits_0^{\pi} d\vartheta' b(\vartheta') \sum_{k=1}^{n} \left[1 + \frac{\sin^2\vartheta}{(\cos\vartheta - \cos\vartheta')(\cos\vartheta - \cos\vartheta_k)}\right] \tag{3.5}$$

The expression in the brackets in (3.5) may be written as

$$[\ \ ] = \frac{\sin^2\vartheta'}{(\cos\vartheta' - \cos\vartheta_k)(\cos\vartheta - \cos\vartheta')} - \frac{\sin^2\vartheta_k}{(\cos\vartheta' - \cos\vartheta_k)(\cos\vartheta - \cos\vartheta_k)}$$

Then (3.5) takes the form

$$\frac{1}{\pi}\int\limits_0^{\vartheta} d\vartheta \int\limits_0^{\pi} \frac{b(\vartheta')\sin\vartheta'\, d\ln T_n(\vartheta')}{\cos\vartheta - \cos\vartheta'} + \sum_{k=1}^{n} \frac{F_k}{a} \int\limits_0^{\vartheta} \frac{d\vartheta'}{\cos\vartheta' - \cos\vartheta_k}$$

Thus we obtain for $J(\vartheta)$

$$J(\vartheta) = -nK\vartheta + \frac{1}{\pi}\int\limits_0^{\vartheta} d\vartheta \int\limits_0^{\pi} \frac{b(\vartheta')\sin\vartheta'\, d\ln T_n(\vartheta')}{\cos\vartheta - \cos\vartheta'} \tag{3.6}$$

Substituting (3.6) into (3.3) with account for (1.12), (2.14) and altering the order of integration, we find

$$\int\limits_S = -KE^2\beta_1 - \frac{E^2}{n^2\pi^3}\int\limits_0^{\pi} d\vartheta \left[\int\limits_{\vartheta}^{\pi} b\left(\frac{\vartheta'}{2}\right) d\ln T_n \frac{\vartheta'}{2}\right] \int\limits_0^{2\pi} b\left(\frac{\vartheta_1}{2}\right) \operatorname{ctg}\frac{\vartheta_1 - \vartheta}{2}\, d\ln T_n\left(\frac{\vartheta_1}{2}\right)$$

Substituting this expression into (3.1), we obtain the final formula for the power developed:

$$P_2 = \sigma E^2 [c(G_2 - KG_1) - K\beta_1 + \beta_2]$$

$$G_1 = \frac{1}{\lambda}\int\limits_{\mu}^{\lambda+\mu} b(x)\, dx, \qquad G_2 = \frac{1}{\lambda}\int\limits_{\mu}^{\lambda+\mu} b^2(x)\, dx \tag{3.7}$$

$$\beta_2 = \frac{2}{H}\int\limits_{-\infty}^{\mu} b^2(x)\, dx - \frac{2}{n^2\pi^3}\int\limits_0^{\pi} d\vartheta \left[\int\limits_{\vartheta}^{\pi} b\left(\frac{\vartheta'}{2}\right) d\ln T_n \frac{\vartheta'}{2}\right] \int\limits_0^{2\pi} b\left(\frac{\vartheta_1}{2}\right) \operatorname{ctg}\frac{\vartheta_1 - \vartheta}{2}\, d\ln T_n\left(\frac{\vartheta_1}{2}\right)$$

We note that a singular integral with Hilbert kernel appears in the expression for β_2. This integral can be calculated for an arbitrary function $b(^1/_2\vartheta)$ by expanding the function

$$f(\vartheta) = b\left(\frac{\vartheta}{2}\right) \frac{d\ln T_n(\vartheta/2)}{d\vartheta}$$

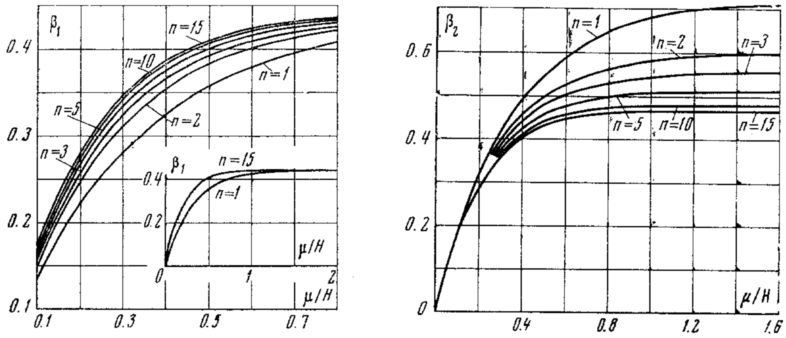

Fig. 3

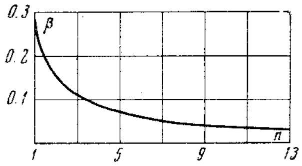

Fig. 4

into a Fourier series. Noting that $f(\nu)$ is an odd function of ϑ, we have

$$f(\vartheta)=\sum_{m=1}^{\infty} b_m \sin m\vartheta, \qquad b_m=\frac{2}{\pi}\int_0^{\pi} b\left(\frac{\vartheta}{2}\right)\sin m\vartheta \, d\ln T_n\left(\frac{\vartheta}{2}\right) \tag{3.8}$$

Substituting (3.8) into (3.7) and performing the integration, using the relation [2]

$$\int_0^{2\pi} \sin m\vartheta' \operatorname{ctg}\frac{\vartheta'-\vartheta}{2}\, d\vartheta' = 2\pi\cos m\vartheta$$

we obtain

$$\beta_2=\frac{2}{H}\int_{-\infty}^{\mu} b^2(x)\,dx-\frac{8}{n^2\pi^3}\sum_{m=1}^{\infty}\frac{C_m^2}{m} \tag{3.9}$$

where

$$C_m=\int_0^{1/2\pi} b(\vartheta)\sin 2m\vartheta \, d\ln T_n(\vartheta)$$

In certain particular cases of magnetic field b(x) specification, the singular integral in β_2 can be calculated directly (see Section 6). Now let us examine some particular cases of baffle location and magnetic field distribution downstream of the electrodes.

4. If the channel has no baffles (n = 1), then (2.9), (2.14), (3.9) take the form

$$\Phi=c+\frac{2\ln 2}{\pi}, \qquad \beta_1=\frac{4}{\pi^2}\int_0^{1/2\pi} b(\psi)\psi\operatorname{ctg}\psi\, d\psi \tag{4.1}$$

$$b(\psi)\equiv b[x(\psi)], \qquad \sin\psi=\exp(\pi x/H)$$

$$\beta_2=\frac{2}{H}\int_{-\infty}^{\mu} b^2(x)\,dx-\frac{8}{\pi^3}\sum_{m=1}^{\infty}\frac{A_m^2}{m}, \qquad A_m=\int_0^{1/2\pi} b(\psi)\operatorname{ctg}\psi\sin 2m\psi\, d\psi$$

Formulas (4.1) coincide with the corresponding formulas of [4], obtained by mapping the upper half-plane $\nu > 0$ onto a rectangle and subsequent solution of the problem by the Fourier method. For a channel with a single pair of baffles (n = 2), the formulas for β_1 and β_2 coincide with the corresponding formulas of [5].

5. Baffles are inserted right up to the electrode zone ($\mu = 0$). In this case it follows from (2.9) that

$$\Phi=c+\frac{2\ln 2}{n\pi} \tag{5.1}$$

This coincides with the corresponding expression of [6]. Since in this case $b(n\theta)$ has the period $\pi/n (T_n(\theta)=\cos n\theta)$, it follows from (2.14) that

$$\beta_1=\frac{4}{n\pi^2}\int_0^{1/2\pi}\left(\frac{\pi}{2}-\theta\right) b(n\theta)\operatorname{tg}(n\theta)\, d(n\theta)=\frac{4}{n\pi^2}\int_0^{1/2\pi} b(\psi)\psi\operatorname{ctg}\psi\, d\psi=\frac{\beta_{10}}{n} \tag{5.2}$$

Similarly, from (3.9)

$$\beta_2 = \beta_{20} / n \tag{5.3}$$

Here β_{10} and β_{20} are the corresponding coefficients for the channel without baffles of width H/n. This result is obvious. In fact, just as was done in [6] for the case V=0, it can be shown that the electric field distribution pattern has the period H/n in y and the straight lines connecting the right ends of the left-hand baffles with the left ends of the corresponding right-hand baffles are equipotentials which divide the entire channel into n identical subchannels of width H/n, connected in series electrically; hence (5.2) and (5.3) follow.

6. Let us consider as an example the simplest case of magnetic field specification downstream of the electrodes

$$b(x) = \begin{cases} 1 & \text{for} \quad 0 < x < \mu \\ 0 & \text{for} \quad x < 0 \end{cases} \tag{6.1}$$

From (2.14) after integration by parts we obtain

$$\beta_1 = \frac{2\mu}{H} - \frac{4}{n\pi^2} \int_0^\theta \ln T_n(\vartheta)\, d\vartheta \left(\theta = \arccos \frac{1}{a}\right) \tag{6.2}$$

Further, using the known formulas for the calculation of the limiting values of the Cauchy type integral, we obtain from (3.7)

$$\beta_2 = \frac{2\mu}{H}\frac{2}{\pi}\arcsin\frac{1}{a} + \left(1 - \frac{2}{\pi}\arcsin\frac{1}{a}\right)\beta_1 + \frac{4}{n^2\pi^3}\sqrt{a^2-1}\int_0^\theta \frac{\ln^2 T_n(\vartheta)\, d\vartheta}{a^2\cos^2\vartheta - 1} + \frac{4}{n^2\pi^3}\sum_{k=1}^{n}\frac{F_k}{a}\int_0^\theta \frac{\ln T_n(\vartheta)\, d\vartheta}{\cos\vartheta - \cos\vartheta_k} \tag{6.3}$$

Figure 3a, b shows the coefficients β_1 and β_2 as a function of μ/H for different values of n, calculated using (6.2) and (6.3).

7. From (6.2) and (6.3) we can obtain the expression for the Joule losses Q in the case in which all the channel walls are insulated, passing to the limit as $\mu/H \to \infty$ $(a \to \infty)$. In this case

$$Q = \sigma E^2 \beta \tag{7.1}$$

where

$$\beta = \lim_{\mu/H\to\infty} (\beta_2 - \beta_1) = \frac{4}{n^2\pi^3}\int_0^1 \frac{\ln^2 T_n(x^{-1})\, dx}{1 - x^2} - \frac{8}{n^2\pi^3}\int_0^1 S_n(x) \ln T_n(x^{-1})\, dx$$

$$S_n(x) = \sum_{k=1}^{n} \frac{|\tau_k| \operatorname{Arcth} |\tau_k|^{-1}}{1 - x\tau_k} \tag{7.2}$$

The values of τ_k are given by (1.5). Figure 4 shows β as a function of the number of baffles, calculated using (7.2). For large values of n (n > 15) $\beta = 2 \ln 2/n\pi$. We note that for n=1 and n=2 (7.2) coincides with the corresponding expressions obtained in [7, 8], with the sole difference that the value of β from (7.2) is twice as large as the value of [7, 8], since the total losses at the inlet to and exit from the magnetic field were taken into account.

8. If the constant-magnitude magnetic field is removed to the distance $\mu + \Delta\mu$ downstream from the electrodes, where $\Delta\mu > 1.3$ H/n, and then the field decays, following any law, the values of the coefficients β_1 and β_2 are found as follows. To find the coefficient β_1 we can assume that the field is constant along the entire channel; then, as shown in [6], the expression for the dimensionless current has the form: (1−K)Φ. Comparing this last expression with (1.26), we find that in the present case $\beta_1 = \Phi - c$. The coefficient

$$\beta_2 = \Phi - c + n\beta_0$$

is found similarly.

Here β_0 are the dimensionless Joule losses in the subchannel of width H/n with insulated walls in the region of magnetic field variation (at the distance $\mu + \Delta\mu$ downstream from the electrodes), which can be found using the formulas obtained in [9]. Thus, from the viewpoint of obtaining the maximal removable power and maximal efficiency the following channel and magnetic field geometry will be optimal: $\mu > 1.3H$, $\Delta\mu > 1.3H/n$. In this case the end zones make maximal contribution to the removable power, and the Joule losses in the zone of nonuniformity of the magnetic field are minimal. (The latter statement is valid only within the framework of two-dimensional theory. As shown in [11], in accounting for the three-dimensional nature of the problem, there arise additional Joule losses which increase with increase of the magnetic field removal downstream from the electrodes, which is associated with closure of the currents through the boundary layer in the plane of the channel cross section.)

Thus we obtain the following expressions for the power $P_1 = UI$, which can be taken from the electrodes and is developed in a channel of electromagnetic power P_2 and electromagnetic efficiency η:

$$P_1 = \sigma E^2 K\,[cG_1 - K\Phi + \beta_1], \quad P_2 = \sigma E^2\,[c\,(G_2 - KG_1) - K\beta_1 + \beta_2]$$

$$\eta = \frac{P_1}{P_2}, \qquad K = \frac{\sigma R(cG_1 + \beta_1)}{1 + \sigma R\Phi}$$

Here R is the load resistance. Expressions (2.14) and (3.9) for the coefficients β_1 and β_2 are only valid, strictly speaking, for the channel with infinitely long electrodes. However, in practice these formulas can be used to calculate the end effect in channels with finite but quite long electrodes, when the mutual influence of the processes at the entrance to and exit from the channel can be neglected. For channels without baffles this holds for any $c > 1$ [4]. Moreover, introduction of baffles into the channel reduces the scale of the electric field nonuniformity zone at the entrance to and exit from the channel, and therefore we can assume that the formulas obtained for β_1 and β_2 are clearly valid for channels with $c > 1$.

In conclusion, we note that all the formulas obtained above are valid for the channel operating in other (pump, decelerator) regimes, as well as in the generator regime [10].

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